Journées ANR COSS

A multi-population traffic flow model on networks accounting for routing strategies

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joint work with Adriano Festa and Fabio Vicini (Polytechnic of Turin, Italy)

Outline of the talk

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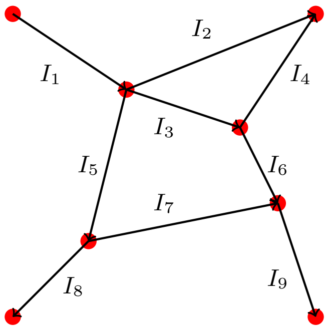
1. Conservation laws on networks
2. A multi-population model on networks accounting for routing strategies
3. Numerical tests

Conservation laws on networks[1]

Networks

Finite collection of directed arcs $I_\ell =]a_\ell, b_\ell[$ connected by nodes



[1] [Holden-Risebro 1995; Garavello-Piccoli 2006]

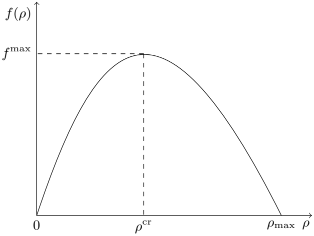
LWR model[2]

Non-linear transport equation: PDE for mass conservation

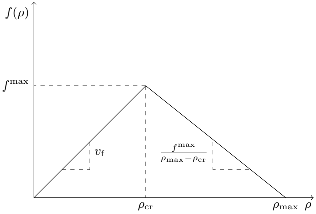
$$\partial_t \rho + \partial_x f(\rho) = 0 \qquad x \in \mathbb{R}, t > 0$$

- $\rho = \rho(t,x) \in [0, \rho_{\max}]$ mean traffic density
- $f(\rho) = \rho v(\rho)$ flux function

Empirical flux-density relation: fundamental diagram



Greenshields '35



Newell-Daganzo

[2] [Lighthill-Whitham 1955; Richards 1956]

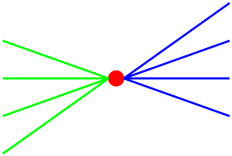
Extension to networks

- LWR on networks:
  [Holden-Risebro, 1995; Coclite-Garavello-Piccoli, 2005; Garavello-Piccoli, 2006]
  - LWR on each road
  - Optimization problem at the junction
- Modeling of junctions with a buffer:
  [Herty-Lebacque-Moutari, 2009; Garavello-Goatin, 2012; Garavello, 2014; Bressan-Nguyen, 2015; LaurentBrouty&al, 2019]
  - Junction described by one or more buffers
  - Suitable for optimization and Nash equilibrium problems

Riemann problem at point junctions

$$\begin{cases} \partial_t \rho_\ell + \partial_x f_\ell(\rho_\ell) = 0 \\ \rho_\ell(0,x) = \rho_{\ell,0} \end{cases}$$

$\ell = 1, \dots, n+m$



Riemann solver: $\mathcal{RS}_J : (\rho_{1,0}, \dots, \rho_{n+m,0}) \longmapsto (\bar\rho_1, \dots, \bar\rho_{n+m})$ s.t.

- conservation of cars: $\sum_{i=1}^{n} f_i(\bar\rho_i) = \sum_{j=n+1}^{n+m} f_j(\bar\rho_j)$
- waves with negative speed in incoming roads
- waves with positive speed in outgoing roads

Consistency condition:

$$\mathcal{RS}_J\big(\mathcal{RS}_J(\rho_{1,0}, \dots, \rho_{n+m,0})\big) = \mathcal{RS}_J(\rho_{1,0}, \dots, \rho_{n+m,0}) \qquad \textbf{(CC)}$$

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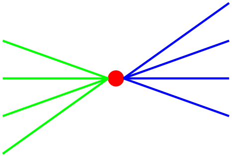

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Set $\bar\gamma_\ell = f_\ell(\bar\rho_\ell)$

Dynamics at junctions

(A) **prescribe a fixed distribution of traffic in outgoing roads**

$$A = \{a_{ji}\} \in \mathbb{R}^{m \times n} : \; 0 < a_{ji} < 1, \sum_{j=n+1}^{n+m} a_{ji} = 1$$

outgoing fluxes $= A\cdot$ incoming fluxes
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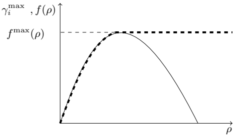
(A)+(B) equivalent to a LP optimization problem which gives a unique solution to RPs (under suitable hypotheses on A)

More incoming than outgoing roads $\Longrightarrow$ priority parameters

Demand & Supply [3]

Incoming roads $i = 1, \dots, n$:

$$\gamma_i^{\max}(\rho_{i,0}) = \begin{cases} f_i(\rho_{i,0}) & \text{if } 0 \le \rho_{i,0} < \rho^{\text{cr}} \\ f_i^{\max} & \text{if } \rho^{\text{cr}} \le \rho_{i,0} \le 1 \end{cases}$$



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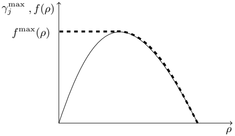

[3] [Lebacque]

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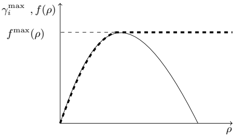

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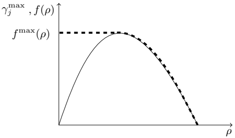


Admissible fluxes at junction: $\Omega_\ell = [0, \gamma_\ell^{\max}]$

[3] [Lebacque]

Priority Riemann Solver[4]

(A) distribution matrix of traffic from incoming to outgoing roads

$$A = \{a_{ji}\} \in \mathbb{R}^{m \times n} : \quad 0 \leq a_{ji} \leq 1, \sum_{j=n+1}^{n+m} a_{ji} = 1$$

(B) priority vector

$$P = (p_1, \ldots, p_n) \in \mathbb{R}^n : \quad p_i > 0, \sum_{i=1}^{n} p_i = 1$$

(C) feasible set

$$\Omega = \left\{ (\gamma_1, \cdots, \gamma_n) \in \prod_{i=1}^{n} \Omega_i : A \cdot (\gamma_1, \cdots, \gamma_n)^T \in \prod_{j=n+1}^{n+m} \Omega_j \right\}$$

[4] [DelleMonache-Goatin-Piccoli, CMS 2018]

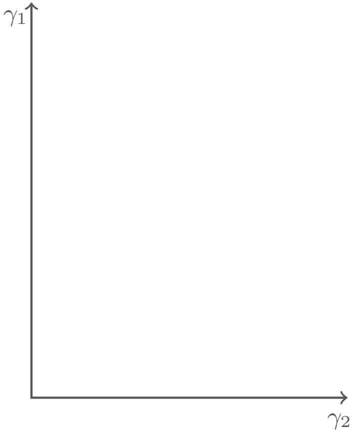
Priority Riemann Solver

Algorithm 1 Recursive definition of $\mathcal{PRS}$

Set $J = \emptyset$ and $J^c = \{1, \dots, n\} \setminus J$.
while $|J| < n$ **do**
  $\forall i \in J^c \to h_i = \max\{h : h\, p_i \le \gamma_i^{max}\} = \frac{\gamma_i^{max}}{p_i}$,
  $\forall j \in \{n+1 \dots, n+m\} \to h_j = \sup\{h : \sum_{i \in J} a_{ji} Q_i + h(\sum_{i \in J^c} a_{ji} p_i) \le \gamma_j^{max}\}$.
  Set $\hbar = \min_{ij}\{h_i, h_j\}$.
  if $\exists\ j$ s.t. $h_j = \hbar$ **then**
    Set $Q = \hbar\, P$ and $J = \{1, \dots, n\}$.
  else
    Set $I = \{i \in J^c : h_i = \hbar\}$ and $Q_i = \hbar\, p_i$ for $i \in I$.
    Set $J = J \cup I$.
  end if
end while

$\mathcal{PRS}$ in practice

2×2 junction ($n = 2$, $m = 2$):



1. Define the spaces of the incoming fluxes

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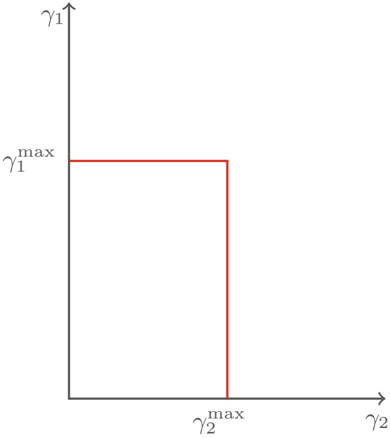

1. Define the spaces of the incoming fluxes
2. Consider the demands

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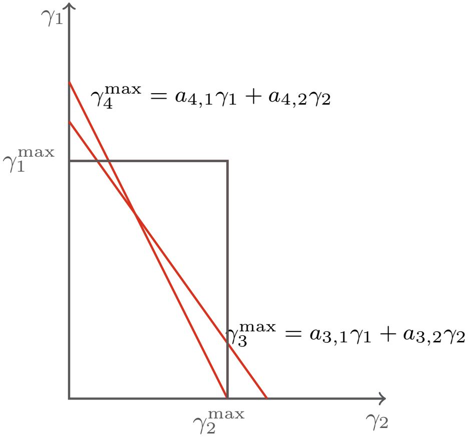

1. Define the spaces of the incoming fluxes
2. Consider the demands
3. Trace the supply lines

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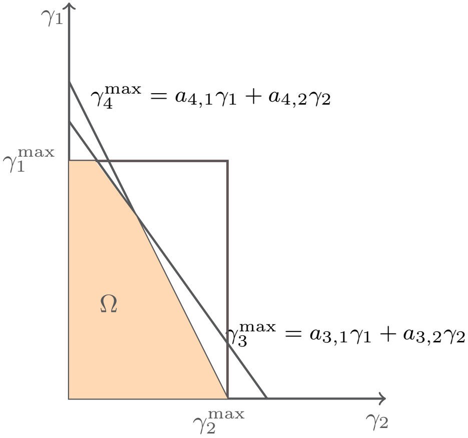

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4. The feasible set is given by Ω

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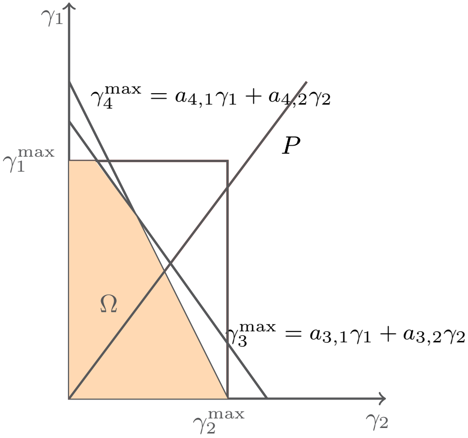

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4. The feasible set is given by Ω
5. Trace the priority line

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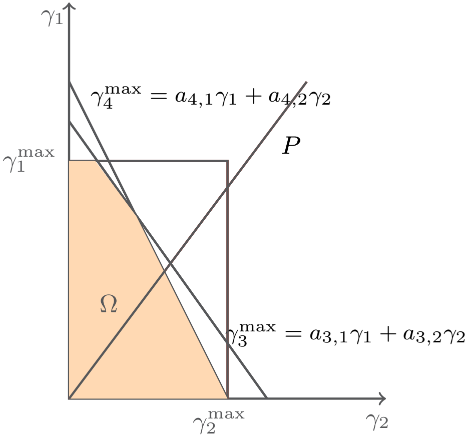

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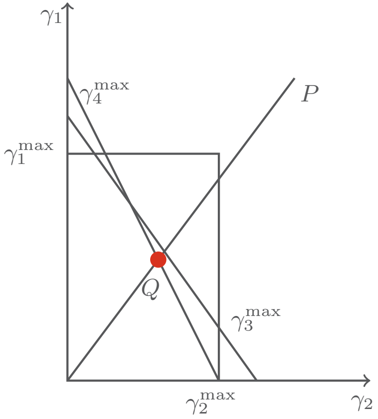
Different situations can occur

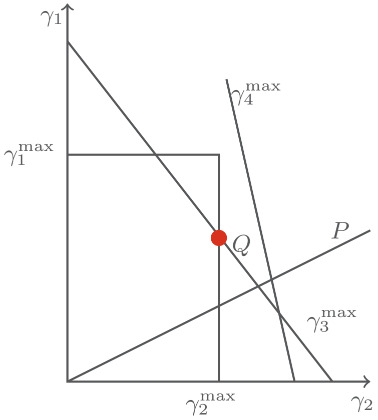
$\mathcal{PRS}$: optimal point

P intersects the supply lines in $\partial\Omega$



$\mathcal{PRS}$: optimal point

P intersects the supply lines outside Ω



$\mathcal{PRS}$

Definition ($\mathcal{PRS}$)

$Q = (\bar{\gamma}_1, \ldots, \bar{\gamma}_n)$ incoming fluxes defined by Algorithm 1
$A \cdot Q^T = (\bar{\gamma}_{n+1}, \ldots, \bar{\gamma}_{n+m})^T$ outgoing fluxes
Set

$$\bar{\rho}_i = \begin{cases} \rho_{i,0} & \text{if } f(\rho_{i,0}) = \bar{\gamma}_i \\ \rho \geq \rho^{\text{cr}} & \text{s.t. } f(\rho) = \bar{\gamma}_i \end{cases} \qquad i \in \{1, \ldots, n\}$$

$$\bar{\rho}_i = \begin{cases} \rho_{j,0} & \text{if } f(\rho_{j,0}) = \bar{\gamma}_i \\ \rho \leq \rho^{\text{cr}} & \text{s.t. } f(\rho) = \bar{\gamma}_j \end{cases} \qquad j \in \{n+1, \ldots, n+m\}$$

Then, $\mathcal{PRS} : [0, \rho_{\max}]^{n+m} \to [0, \rho_{\max}]^{n+m}$ is given by

$$\mathcal{PRS}(\rho_{1,0}, \ldots, \rho_{n+m,0}) = (\bar{\rho}_1, \ldots, \bar{\rho}_n, \bar{\rho}_{n+1}, \ldots, \bar{\rho}_{n+m}) \,.$$

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Remark: $\mathcal{PRS}$ may be obtained as limit of solvers defined by Dynamic Traffic Assignment based on junctions with queues
[Bressan-Nordli, NHM, 2017]

Cauchy problem: existence results

Theorem (DelleMonache-Goatin-Piccoli, CMS 2018)

If a Riemann solver satisfies (P1)-(P3), then every Cauchy problem with BV initial data admits a weak solution.

Proof: Wave-Front Tracking, bound on $\mathrm{TV}(f)$ and "big shocks".

Proposition (DelleMonache-Goatin-Piccoli, CMS 2018)

The Priority Riemann Solver $\mathcal{PRS}$ satisfies (P1)-(P3) for junctions with $n \leq 2$, $m \leq 2$ and $0 < a_{ji} < 1$ for all i, j.

$\mathcal{PRS}$: summary

General Riemann Solver at junctions:

- no restrictions on A
- no restrictions on the number of roads
- priorities come before flux maximization
- compact algorithm to compute solutions
- general existence result via Wave-Front-Tracking

Outline of the talk

Multi-class model on networks[5]

ρ_ℓ^c density of vehicles of class $c = 1, \ldots, N_c$ on link I_ℓ
$\rho_\ell = \sum_c \rho_\ell^c$ total traffic density on link I_ℓ

$$\partial_t \rho_\ell^c + \partial_x(\rho_\ell^c v_\ell(\rho_\ell)) = 0 \qquad x \in I_\ell, t > 0,$$

[5] [Garavello-Piccoli, CMS 2005; Cristiani-Priuli, NHM 2015; Samanayarake&al, Tr. Sci. 2018]

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Summing on $c = 1, \ldots, N_c$ we get

$$\partial_t \rho_\ell + \partial_x(\rho_\ell v_\ell(\rho_\ell)) = 0 \qquad x \in I_\ell, t > 0,$$

[5] [Garavello-Piccoli, CMS 2005; Cristiani-Priuli, NHM 2015; Samanayarake&al, Tr. Sci. 2018]

Multi-class junction conditions

1. *Compose the total distribution matrix.*
   $A^c = \left\{a^c_{ji}\right\}_{i,j}$ distribution matrices for each class $c = 1, \ldots, N_c$. Set

   $$A := \{a_{ji}\}, \quad \text{where} \quad a_{ji} := \sum_{c=1}^{N_c} a^c_{ji} \frac{\rho^c_i}{\rho_i} \tag{1}$$

   weighted distribution matrix for the *total density* of the populations at the junction.
2. *Compute the fluxes* $(\bar{\gamma}_1, \ldots, \bar{\gamma}_{n+m})$
   using the selected Riemann solver $\mathcal{RS}_J = \mathcal{RS}^A_J$ corresponding to (1).
3. *Distribute the fluxes among the various classes.*
   The incoming and outgoing fluxes for each class are given by

   $$\bar{\gamma}^c_i = \frac{\rho^c_i}{\rho_i}\bar{\gamma}_i, \quad i = 1, \ldots, n, \quad \bar{\gamma}^c_j = \sum_{i=1}^{n} a^c_{ji}\bar{\gamma}^c_i, \qquad j = n+1, \ldots, n+m.$$

Strategy modeling on network (static)

Goal: minimize the weighted distance from the target $\mathcal{T}^c$

Value function

$$u_\ell^c(y) = \inf\left\{d_c(y,x) \colon x \in \mathcal{T}^c\right\}$$

where

$$d_c(y,x) = \inf\left\{\int_y^{L_\ell} \frac{1}{g^c(\rho_\ell(z,t))}dz + \sum_i \int_0^{L_i} \frac{1}{g^c(\rho_{\ell_i}(z,t))}dz\right\}$$

g^c being the *running cost*, thus giving the “shortest path”

Strategy modeling on network (cont'd)

Weighted distance from the target $\mathcal{T}^c$: u_ℓ^c viscosity solution of

$$\begin{cases} \partial_x u_\ell^c + \frac{1}{g^c(\rho_\ell(x,t))} = 0 & x \in I_\ell, t > 0, \quad \textbf{(static)} \\ \min\limits_{\ell \in Out(J_k)} u_\ell^c(t,0) = u_\ell^c(t,L_l) & J_k \in \mathcal{J} \setminus \mathcal{T}^c,\ l \in Inc(J_k) \\ u_\ell^c(L_\ell) = 0\,, & \pi_\ell(L_\ell) \in \mathcal{T}^c \end{cases}$$

where g^c is the running cost ($g^c \equiv 1$ or $g^c = v_\ell$)

$\longrightarrow$ **eikonal equation on network**

[Schieborn-Camilli 2013; Camilli-Festa-Schieborn 2013]

Strategy modeling on network (cont'd)

Weighted distance from the target $\mathcal{T}^c$: u^c_ℓ viscosity solution of

$$\begin{cases} \partial_x u^c_\ell + \frac{1}{g^c(\rho_\ell(x,t))} = \partial_t u^c_\ell & x \in I_\ell, t > 0, \quad \textbf{(dynamic)} \\ \min\limits_{\ell \in Out(J_k)} u^c_\ell(t,0) = u^c_\ell(t, L_l) & J_k \in \mathcal{J} \setminus \mathcal{T}^c,\ l \in Inc(J_k) \\ u^c_\ell(L_\ell) = 0\,, & \pi_\ell(L_\ell) \in \mathcal{T}^c \end{cases}$$

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where g^c is the running cost ($g^c \equiv 1$ or $g^c = v_\ell$)

$\longrightarrow$ **eikonal equation on network**
[Schieborn-Camilli 2013; Camilli-Festa-Schieborn 2013]

We set

$$\bar{u}^c = \min_{j \in Out(J_k)} u_j^c(0)$$

and

$$\alpha_{ji}^c = \frac{\psi(u_j^c(t,0) - \bar{u}^c)}{\sum_{z \in Out(J_k)} \psi(u_z^c(t,0) - \bar{u}^c)}$$

with ψ *activation function*
(e.g. $\psi(x) = \chi_{]-\infty,0]}(x)$ or $\psi(x) = 1/(1 + e^{-\epsilon(S-2x)})$)

System discretization

Conservation laws:

$$\rho_{\ell,1}^{c,\nu+1} = \rho_{\ell,1}^{c,\nu} - \frac{\Delta t}{\Delta x_{\ell,1}} \left(\frac{\rho_{\ell,1}^{c,\nu}}{\rho_{\ell,1}^{\nu}} F_{\ell,1}^{\nu} - \bar{\gamma}_{\ell,1}^{c,\nu} \right)$$

$$\rho_{\ell,h}^{c,\nu+1} = \rho_{\ell,h}^{c,\nu} - \frac{\Delta t}{\Delta x_{\ell,h}} \left(\frac{\rho_{\ell,h}^{c,\nu}}{\rho_{\ell,h}^{\nu}} F_{\ell,h}^{\nu} - \frac{\rho_{\ell,h-1}^{c,\nu}}{\rho_{\ell,h-1}^{\nu}} F_{\ell,h-1}^{\nu} \right)$$

$$\rho_{\ell,N_\ell}^{c,\nu+1} = \rho_{\ell,N_\ell}^{c,\nu} - \frac{\Delta t}{\Delta x_{\ell,N_\ell}} \left(\bar{\gamma}_{\ell,N_\ell}^{c,\nu} - \frac{\rho_{\ell,N_\ell-1}^{c,\nu}}{\rho_{\ell,N_\ell-1}^{\nu}} F_{\ell,N_\ell-1}^{\nu} \right)$$

where

$F_{\ell,h}^{\nu} = F_\ell(\rho_{\ell,h}^{\nu}, \rho_{\ell,h+1}^{\nu}) := \min \left\{ D_\ell(\rho_{\ell,h}^{\nu}), S_\ell(\rho_{\ell,h+1}^{\nu}) \right\}$ (Godunov scheme)

$\Delta t \leq \min_{\ell,h} \Delta x_{\ell,h} / V_\ell$ (CFL condition)

Eikonal equations:

$$\frac{u_{\ell,h+1}^{c,\nu} - u_{\ell,h}^{c,\nu}}{\Delta x_{\ell,h}} + \frac{1}{g^c(\rho_{\ell,h}^{\nu})} = 0$$

$$u_{\ell,N_\ell}^{c,\nu} = \min_{i \in Out(J_k)} u_{i,1}^{c,\nu}, \quad x_{\ell,N_\ell} = J_k \in \mathcal{J}$$

System discretization (cont'd)

Junction coupling conditions:

$$\bar{u}^{c,\nu} = \min_{i \in Out(J_k)} u^{c,\nu}_{i,1}$$

$$a^{c,\nu}_{ji} = \frac{\psi(u^{c,\nu}_{j,1} - \bar{u}^{c,\nu})}{\sum_{z \in Out(J_k)} \psi(u^{c,\nu}_{z,1} - \bar{u}^{c,\nu})}$$

$$A^{\nu}_k = \left\{ \sum_{c=1}^{N_c} a^{c,\nu}_{ji} \frac{\rho^{c,\nu}_{i,N_i}}{\rho^{\nu}_{i,N_i}} \right\}_{ji}$$

$$(\bar{\gamma}^{\nu}_{\ell_1}, \ldots, \bar{\gamma}^{\nu}_{\ell_{n_k+m_k}}) = \mathcal{RS}_{A^{\nu}_k}(\rho^{\nu}_{\ell_1}, \ldots, \rho^{\nu}_{\ell_{n_k+m_k}})$$

$$\bar{\gamma}^{c,\nu}_{i,N_i} = \frac{\rho^{c,\nu}_{i,N_i}}{\rho^{\nu}_{i,N_i}} \bar{\gamma}^{\nu}_i, \quad i \in Inc(J_k),$$

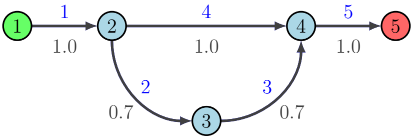
$$\bar{\gamma}^{c,\nu}_{j,1} = \sum_{i=\ell_1}^{\ell_{n_k}} a^{c,\nu}_{ji} \bar{\gamma}^{c,\nu}_i, \quad j \in Out(J_k),$$

Initial and boundary conditions

$$\rho^{c,0}_{\ell,h} = \frac{1}{\Delta x_{\ell,h}} \int_{x_{\ell,h}}^{x_{\ell,h+1}} \bar{\rho}^c_{\ell}(x)dx, \qquad u^{c,\nu}_{\ell,N_\ell} = \rho^{c,\nu}_{\ell,N_\ell} = 0, \quad x_{\ell,N_\ell} \in \mathcal{T}^c,$$

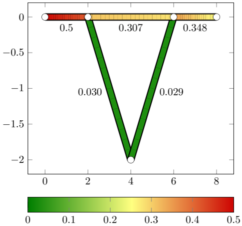
Outline of the talk

Test 1: simple network with one populatiion

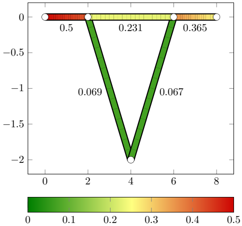


- We assume that the network is empty at $t = 0$, we consider a single population with constant inflow $\phi(t) = 0.5$ at node 1 for $t \in [0, 6]$ and we set $\phi(x) = 0$ for $t \in]6, 12]$.
- The space and time meshes are set to $\Delta x = 0.05$ and $\Delta t = 0.01$, respectively.
- If the agents are *simply-informed*, i.e. $g(\rho) \equiv 1$, the whole population follows the path 1-2-4-5, since it is the shortest path to destination.
- Smooth activation function $\psi(x) = 1/(1 + e^{-(S-2x)})$

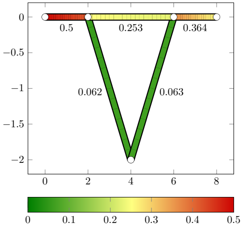
Test 1: results



(a) Simply-informed



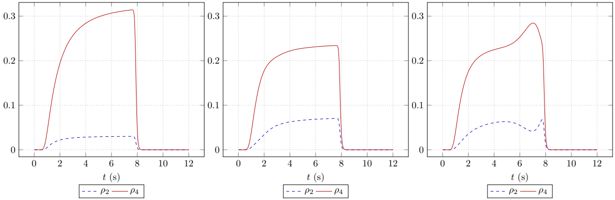
(b) Informed



(c) Highly-informed

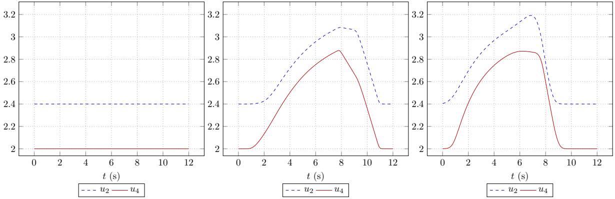
Figure: Simple network with 5 roads; ρ at $t = 6$ sec is depicted; $\max_{x \in I_\ell} \rho_\ell(x, 6)$ is reported below the streets.

Test 1: results



(a) Simply-informed (b) Informed (c) Highly-informed

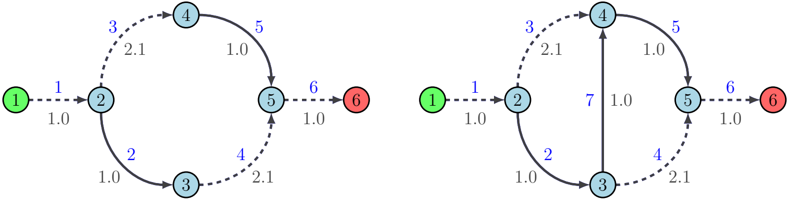
Figure: Time evolution of the density $\rho_2(t,0)$ and $\rho_4(t,0)$ at intersection 2



(a) Simply-informed (b) Informed (c) Highly-informed

Figure: Time evolution of the costs $u_2(t,0)$ and $u_4(t,0)$ at intersection 2.

Test 2: Braess' Paradox

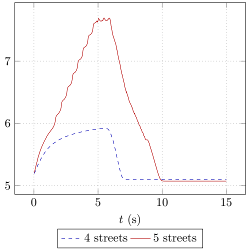


(a) 4 streets (b) 5 streets

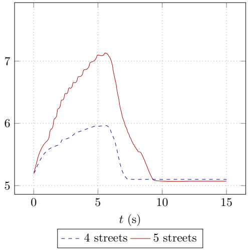
- We take a constant inflow $\phi(t) = 0.5$ at the junction 1 for $t \in [0, 6]$, we set $\phi(t) = 0$ for $t \in]6, 15]$
- The space and time meshes are set to $\Delta x = 0.05$ and $\Delta t = 0.01$.

Test 2: results

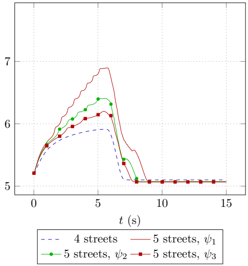
We compare the performances of the two networks in the terms of travel time $TT(1,6,t)$ for $t \in [0,12]$, i.e. the time needed by a single vehicle starting at node 1 at time t to reach the destination node 6.



(a) Simply-informed



(b) Informed



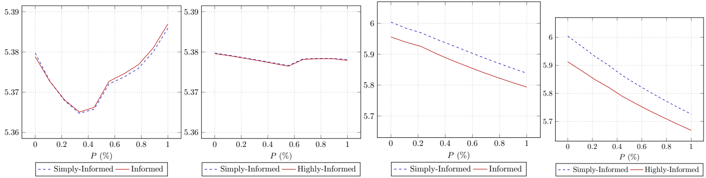
(c) Highly-informed

Figure: Travel time $TT(1,6,t)$, $t \in [0,15]$, for different levels of information. Here $\epsilon = \{1,2,3\}$

Test 3: two populations

- We consider again the Braess' 4-roads and 5-roads networks.
- We compare the behaviour of two populations with a different information level: we compare their mean travel times (MTT) on the interval $[0, T]$:

$$MTT(x_i, x_j) = \frac{1}{T} \sum_{k=1}^{[T/\Delta t]} TT(x_i, x_j, k\Delta t). \tag{2}$$



(a) 4 streets; Simply-informed VS Informed

(b) 4 streets; Simply-informed VS Highly-informed

(c) 5 streets; Simply-informed VS Informed

(d) 5 streets; Simply-informed VS Highly-informed

Figure: MTT (1,6) depending on the populations ratio $P \in [0, 1]$.

Conclusion

- A. Festa and P. Goatin, Modeling the impact of on-line navigation devices in traffic flows, 2019 IEEE 58th Conference on Decision and Control (CDC), Nice, France (2019), 323-328.
- A. Festa, P. Goatin and F. Vicini, Navigation system based routing strategies in traffic flows on networks, submitted

Multi-population model accounting for routing choices:

- Can be applied to any Riemann Solver at junction
- Solves eikonal equations on networks
- Reproduces expected behaviour
- Can be extended to route choice based on traffic forecast
- Convergence?

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